The Causal Theory revisited

Review of *Quantum Causality*, by Peter J. Riggs. Springer. 2009.

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While standard quantum theory is empirically extremely successful, it is a measurement theory, making predictions about possible outcomes of measurements. Since the notion of measurement is rather ambiguous, it can not be regarded as a fundamental theory of nature. An alternative theory which is free of this problem and yet reproduces the predictions of standard quantum theory (at least when those are unambiguous) is the de Broglie-Bohm theory. This theory forms the subject of Riggs' book "Quantum Causality".

The book is centered around some novel elements, of which I think the most important ones are: a new notion of energy, an alternative view on the nature of the wavefunction, a proposal for experimentally distinguishing the de Broglie-Bohm theory from standard quantum theory and an attempt to derive the exclusion principle. I want to critically assess these elements here.

The de Broglie-Bohm theory, or Causal Theory as Riggs prefers to call it, is a theory about particles that follow trajectories in a deterministic way, influenced by the wavefunction. The possible trajectories $\mathbf{x}_k(t)$ of the particles are solutions to the guidance equation

$$\frac{d\mathbf{x}_k}{dt} = \frac{1}{m_k}\boldsymbol{\nabla}_k S\,, \tag{1}$$

where S is the phase of the wavefunction, i.e. $\psi = |\psi|e^{iS/\hbar}$. The wavefunction itself satisfies the Schrödinger equation. The theory was originally presented by de Broglie in 1927 and rediscovered and further developed by Bohm in 1952. Bohm presented the theory in a different way than de Broglie. While de Broglie regarded the guidance equation as the fundamental equation of the theory, Bohm suggested to regard a Newtonian, second-order differential equation as fundamental. This equation is found by taking the time derivative of the guidance equation, viz.,

$$\frac{d^2\mathbf{x}_k}{dt^2} = -\frac{1}{m_k}\boldsymbol{\nabla}_k(V+Q)\,, \tag{2}$$

where V is the potential that appears in the Hamiltonian of the Schrödinger equation and

$$Q = -\sum_k \frac{\hbar^2}{2m_k}\frac{\nabla_k^2|\psi|}{|\psi|} \tag{3}$$

is the so-called quantum potential. In addition, Bohm assumed the guidance equation as a constraint on the possible momenta, a constraint that in principle could be relaxed.

Riggs also regards the quantum potential as an essential ingredient of the de Broglie-Bohm theory. But unlike Bohm, his motivation for this is not based on possible modifications of the theory. Instead, Riggs claims that the quantum potential formulation allows for a causal description of phenomena unlike the formulation based on the guidance equation. Riggs states that accounts based on the guidance equation "fall short of a full causal explanation of quantum behaviour because such accounts are primarily *kinematic* descriptions" (p. 95), without further explanation. However, this is not correct. The formulation based on the guidance equation allows for the type of causal description envisioned by Riggs just as well as the quantum potential formulation. The only difference is that the guidance equation is an Aristotelian force law, instead of a Newtonian one, with $\boldsymbol{\nabla}_k S$ playing the role of an Aristotelian force, expressing the effect of the wavefunction on the particle.

Riggs' secondary reason for adhering to the quantum potential is its alleged role in understanding the notion of energy in the de Broglie-Bohm theory. However, Riggs' notion of energy differs from the usual one. For a single particle system, the usual definition for the energies of the particle and the wave for a trajectory $(\mathbf{x}(t), \psi(\mathbf{x}, t))$ read

$$E_p = \left(\frac{m}{2}\boldsymbol{\nabla}S(\mathbf{x},t)\cdot\boldsymbol{\nabla}S(\mathbf{x},t) + V(\mathbf{x}) + Q(\mathbf{x},t)\right)\Big|_{\mathbf{x}=\mathbf{x}(t)} \tag{4}$$

and

$$E_w = \int d^3x\psi^*(\mathbf{x},t)\left(-\frac{1}{2m}\nabla^2 + V(\mathbf{x})\right)\psi(\mathbf{x},t)\,. \tag{5}$$

While the energy of the wavefunction is always conserved, the energy of the particle is in general not. This non-conservedness is the main reason for Riggs to suggest an alternative definition of energy. According to this definition, E_w is not the energy of the wave alone, but of the wave and particle together. The energy of the wave itself is then defined as this total energy minus the kinetic energy of particle. As such, the energy is indeed conserved. However, these definitions seem to imply some strange properties. First of all, the total energy does not depend on the particle. Second, the energy of the wave depends on the particle and hence, while the evolution of the wavefunction is not influenced by the particle, the wavefunction can nevertheless exchange energy with the particle. But despite of these apparent unnatural properties, the question arises what is gained by introducing this notion of energy. Riggs applies his definition to a couple of situations, displaying in detail how these energies change. But in my opinion these examples do not illustrate the possible naturalness or usefulness of those definitions. Instead their invocation rather seems to complicate the description of those situations. The insistence on conservation of energy is also not motivated well enough. Riggs does not seem to question its general validity, despite the difficulties with it that are already present in general relativity [1].

In any case, while the quantum potential formulation and associated notions such as energy can be useful in certain cases, for example in studying the classical limit, they are in general unnecessary and rather tend to complicate matters. This has been noted several times before (see for example [2] for a thorough discussion).

Riggs also holds an unconventional view on the wavefunction (see section 3.6.2). In the de Broglie-Bohm theory, the wavefunction is a function on configuration space $\mathbb{R}^{3N}$, with N the number of particles, and not on physical space $\mathbb{R}^3$. This is one of the important features of the theory, it gives rise to entanglement and is key to quantum nonlocality. Riggs acknowledges this, but wants to regard the wavefunction rather as a mathematical representation of a wave in physical space, which he calls the *wave field*. Riggs finds his motivation in observed wave-like properties in particular experiments (see section 4.5). However, while these experiments indeed seem to call for a wave-like object, it is not clear to me why they should be regarded as indicative for a wave in physical space rather than configuration space. Quantum correlations namely seem to suggest otherwise. Nevertheless, it is interesting to consider the question whether the de Broglie-Bohm theory can be reformulated in terms of a wave (or waves) in physical space. Such a program has been considered in detail in the past by, amongst others, Einstein, de Broglie and Vigier (see [3, 4] for some references). However, to my knowledge, this program was not very successful. The program was recently revived by Norsen [5], who succeeded in constructing such a theory. Nevertheless this theory looks a bit extravagant and is hard to take serious (as Norsen himself acknowledges). However, Riggs himself does not make concrete how those waves in physical space should be described mathematically (nor mentions the existing work on this). As such, I find it regrettable that Riggs keeps referring to those objects throughout the book.

These waves in physical space play an important role in Chapter 6, where an argument is advanced to suggest that by considering waves in physical space the exclusion principle could be derived. However, I find the argument very weak. The first part of the argument (in section 6.4) is obscure. Without giving a definition of the waves in physical space, Riggs argues for the appearance of a superposition of such waves with a relative minus sign. This minus sign is seen as the origin of the antisymmetry of a fermionic two-particle system. However, that minus sign seems to have nothing to do with possible permutation symmetry. In the second part (in section 6.5), Riggs makes an attempt to be more concrete. However, as admitted by Riggs, the wavefunction that is obtained is not anti-symmetric under permutation of the particle labels and hence the derivation of the exclusion principle fails. (Actually, there is a further flaw here. The wavefunction Ψ_T on page 178 is supposed to be a wave field, i.e. a wave on physical space, but it is clearly not. It is a function on configuration space.) Riggs blames this failure on the wrong definition of physical waves. This may be so. However, given this negative result, it seems inappropriate to spend a whole chapter (or maybe even anything at all) on this topic.

Another issue that I want to criticize is Riggs' suggested experiment that could possibly empirically distinguish between the de Broglie-Bohm-theory and standard quantum theory (see section 5.9). The experiment Riggs has in mind concerns a quantum particle in an infinite potential well, in the ground state. According to the de Broglie-Bohm-theory the actual particle will stand still. On the other hand, according to standard quantum theory, when a momentum measurement is performed a definite momentum will be found. As already explained by Bohm in his seminal paper, this apparent contradiction is easily resolved by taking into account the actual measurement in the de Broglie-Bohm-description of the situation. However, Riggs claims that the theories might yield different predictions, for a particular quantum optics realization of this situation. From his description of the realization, it is not exactly clear to me how Riggs hopes to achieve this discrepancy. However, whatever Riggs' reasoning, it should be clear that such a discrepancy can not be achieved in quantum equilibrium, i.e. when the distribution of particles is given by $|\psi|^2$ over an ensemble of systems with (effective) wavefunction ψ. Only in quantum non-equilibrium, the de Broglie-Bohm theory might yield different predictions than standard quantum theory, but this is not what Riggs seems to have in mind.

To conclude, in the preface Riggs warns the reader that much of what is argued for will be controversial, but expresses the hope that the arguments will engender some lively discussions. As my above reservations and objections probably convey, I agree with the former, but I am afraid that Riggs' arguments are not strong enough to ensure the latter.

References

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